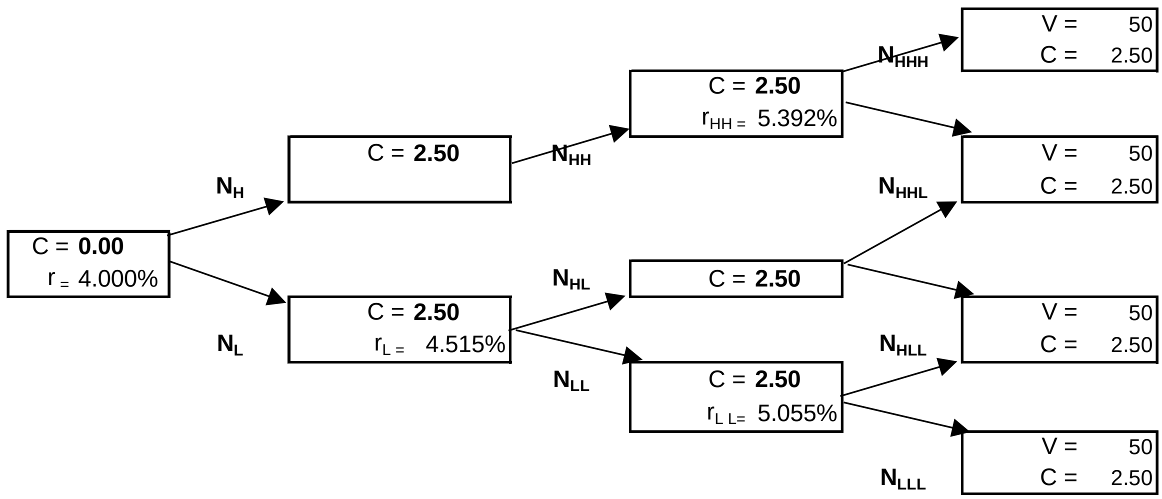
Questions from the 2000 Exam:

32. (3 points)

You are given the following partially completed annual binomial table.



According to Fabozzi, The Handbook of Fixed income Securities, calculate the value of the bond if it is putable at par ($50) in years one and two.

Solutions to questions from the 2000 Exam:

Question 32
To calculate the value of the bond (putable at par ($50) in years one and two) we need to:

a. Determine the Value at a Node (N_{node})

A bond's value at a given node depends on the bond's value at the two nodes to its right. A bond's value at a given node depends on future cash flows from:

1. the bond's value one period from now.
2. coupon payment cash flows.

Let:

V_H = The bond's value along the upper path.
V_L = The bond's value along the lower path.
C = The coupon payment.

Thus, a bond's value at node N is given by $N = \frac{1}{2}\left(\frac{V_H + C}{1+r^*} + \frac{V_L + C}{1+r^*}\right)$, where r* is the 1 year discount rate.

b. Determine the interest rates at the nodes in question.

The relationship between r_L *(t*he lower one-year rate one year forward) and r_H (the higher one-year rate one year forward) is as follows: $r_H = r_L e^{2\sigma}$, where σ= assumed volatility of the one-year rate. Keep in mind that $r_{HH} = r_{LL}e^{4\sigma}$ and $r_{HL} = r_{LL}e^{2\sigma}$

- Solve for σ. Thus, $5.932\% = 5.055\% * e^{4\sigma}$; $\sigma = [\ln(5.932\% / 5.055\%)] / 4 = .04$
- Thus, $r_{HL} = 5.055\% e^{2\sigma} = 5.055\% * 1.083 = 5.476\%$; $r_H = 4.515\% * e^{2\sigma} = 4.515 * * 1.083 = 4.891\%$

c. 1. Compute the bonds values at V_{HH} and at V_{HL} and V_{LL}:

The put option will be exercised by the holder if the bond's value falls below par. Thus, if the value of the computed price at any node is less than 50, it must be replaced by 50 to determine the value of the put option.

$$V_{HH} = \max([\frac{V_{HHH} + V_{HHL} + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50+50+5)}{2*(1.05932)}, 50] = \max(49.56, 50) = 50$$

$$V_{HL} = \max([\frac{V_{HHL} + V_{HLL} + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50+50+5}{2*(1.05476)}, 50] = \max(49.77, 50) = 50$$

$$V_{LL} = \max([\frac{V_{HLL} + V_{LLL} + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50+50+5}{2*(1.05055)}, 50] = \max(49.97, 50) = 50$$

2. Compute the bonds values at V_H and at V_L and V:

$$V_H = \max([\frac{V_{HH} + V_{HL} + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50+50+5)}{2*(1.04891)}, 50] = \max(50.051, 50) = 50.051$$

$$V_L = \max([\frac{V_{HL} + V_{LL} + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50+50+5}{2*(1.04515)}, 50] = \max(50.232, 50) = 50.232$$

$$V = \max([\frac{V_H + V_L + 2C)}{2*(1+r^*)}, 50] = \max([\frac{50.05+50.23+5}{2*(1.04)}, 50] = \max(50.62, 50) = 50.62$$

See pages 781 through 784.