Solutions to questions from the 2006 exam

20. (2 points) A currency swap has a remaining life of 15 months. The swap involves exchanging annual euro interest for dollar interest. The principal amounts are also exchanged at the end of the life of the swap. You are given the following additional information:
    - The swap involves exchanging interest at 11% on €25 million for interest at 8% on $30 million once a year.
    - The term structure of interest rates in both Europe and the United States is currently flat.
    - If the swap were negotiated today, the interest rates exchanged would be 8% in euros and 6% in dollars.
    - All interest rates are quoted with annual compounding.
    - The current exchange rate (dollars per euro) is 1.25.

    Calculate the value of the swap to the party paying dollars.

    SHOW ALL WORK.

Initial comments:

A currency swap can be valued as the difference in positions taken in two bonds (absent default risk).

*If V_{swap} is defined as the value in U.S. dollars of a swap where **dollars are received** and a **foreign currency is paid**, then*

$$V_{swap} = B_D - S_0 B_F, \text{ where}$$

B_F is the value, measured in the foreign currency, of the foreign-denominated bond underlying the swap

B_D is the value of the U.S. dollar bond underlying the swap, and

S_0 is the spot exchange rate (i.e. the number of units of domestic currency per unit of foreign currency).

The value of a swap can therefore be determined from:

LIBOR rates in the two currencies

The term structure of interest rates in the domestic currency, and

The spot exchange rate.

*The value of a swap where the **foreign currency is received** and **dollars are paid** is*

$$V_{swap} = S_0 B_F - B_D$$

***Example** (value of a swap where the **foreign currency is received** and **dollars are paid**)*

A financial institution enters into a 3 year currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year. Assume the following:

- *The term structure of interest rates is flat in both Japan and the United States.*
- *The Japanese rate is 4% per annum and the U.S. rate is 9% per annum (both with continuous compounding).*
- *The principals in the two currencies are $10 million and 1,200 million yen.*
- *The spot exchange rate is 110 yen = $1.*

Step 1: Compute B_D, the value of the U.S. dollar bond underlying the swap:

*The cash flows paid by the institution on the US bond are .08*10M, .08*10M, .08*10M + 10M*

Thus, $B_D = 0.8e^{-0.09 \times 1} + 0.8e^{-0.09x2} + 10.8e^{-0.09x3} = 9.644$ million dollars

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Step 2: Compute B_F, the value of the foreign-denominated (FD) bond underlying the swap:

*The cash flows received under the FD bond are .05*1200M, .05*1200M, .05*1200M + 1200M*

Thus, $B_F = 60e^{-0.04 \times 1} + 60e^{-0.04x2} + 1{,}260e^{-0.04x3} = 1{,}230.55$ million yen

Therefore, the value of the swap in dollars is $\frac{1{,}230.55}{110}$ -9.644 = 1.543million

*Note: If the financial institution had been **paying yen** and **receiving dollars**, the value of the swap would have been -$1.543 million.*

Solution to question 20:

Note: As stated in the problem, all interest rates are quoted with annual compounding.

Step 1: Write an equation to determine the value of the swap to the party paying dollars.

$V_{swap} = S_0B_€ - B_\$$, which values the swap in a way similar to the difference in positions taken in two bonds:

Step 2: Compute $B_€$, the value of a Euro bond underlying the swap:

*The cash flows received under the Euro bond are .11*25M€* at 3 mos., *and [.11*25M€ + 25M€]* at 15 mos.

Thus, $B_€ = 2.75M€ /(1.08)^{3/12} + 27.75M€/(1{,}08)^{15/12} = 27{,}902{,}397$ Euro

Step 3: Compute $B_\$$, the value of the U.S. dollar bond underlying the swap:

*The cash flows paid by on the US bond are .08*30M, at 3 mos., and [.08*30M$ + 30M$]* at 15 mos.

Thus, $B_\$ = 2.4M\$ /(1.06)^{3/12} + 32.4M\$/(1.06)^{15/12} = 32{,}489{,}295$ dollars

Step 4: Using the equation in Step 1, and the results from Steps 2 and 3, compute the value of the swap to the party paying dollars.

*$V_{swap} = 27{,}902{,}397 * (1.25) - 32{,}489{,}295 = \$2{,}388{,}701$*