Questions from the 2007 Exam:

7. (2 points) A collection of general liability policies has the following properties:

- Annual pure premium trend of 11.1% impacts each loss uniformly.
- The 2007 expected value pure premium for a $1,000,000 excess of $500,000 contract is $4,000.
- The 2007 basic limit is $250,000.
- The 2006 increased limits table, where I(K) represents the ILF at limit K, is given by:

K	I(K)
225,000	0.94
250,000	1.00
450,000	1.40
500,000	1.48
900,000	1.90
1,000,000	1.96
1,350,000	2.15
1,500,000	2.18

Calculate the 2007 basic limits pure premium for this collection of policies.

Supporting materials. Our solution to question 6 from the 2005 Exam.

Questions from the 2005 Exam:

6. (2 points) Given the following table of increased limit factors:

Policy Limit	Increased Limit Factor
$95,238	0.956
$100,000	1.000
$105,000	1.045
$476,190	1.600
$500,000	1.632
$525,000	1.660
$1,904,762	2.172
$2,000,000	2.205
$2,100,000	2.235
$2,380,952	2.304
$2,500,000	2.325
$2,625,000	2.333

Assume:

- Annual inflation rate = 5%
- Average loss frequency = 0.20
- Expected severity at the $100,000 limit = $16,000

a. (0.5 points) Calculate the increased limit factor adjusted for one year of inflation for a policy limit of $500,000.

b. (1.5 points) Calculate the pure premium charge adjusted for one year of inflation for a policy covering losses in the layer $2,000,000 excess of $500,000.

Supporting materials. Our solution to question 6 from the 2005 Exam.

Solutions to questions from the 2005 Exam:

Question 6 – Model Solution 1

a. Calculate the increased limit factor adjusted for one year of inflation for a policy limit of $500,000.

Initial comments: Expected value ILFs can be adjusted for inflation using the following formula:

$$I_1(k) = \frac{E[g(x';k)]}{ABLS_1} = \frac{a*E[g(x;k/a)]}{a*E[g(x;b/a)]} = \frac{I(k/a)}{I(b/a)}.$$

Thus, I(500,000/1.05) ÷ I(100,000/1.05) = I(476,190) ÷ I(95,238) = 1.600/.956 = 1.674

b. Calculate the pure premium charge adjusted for one year of inflation for a policy covering losses in the layer $2,000,000 excess of $500,000.

Initial comments:

Excess of Loss pure premium $= E[h(x;r;j)]*E[n] = \{\frac{E[g(x;s)]}{ABLS} - \frac{E[g(x;r)]}{ABLS}\}*[ABLS]*E[n].$

$= [I(s) - I(r)]*[ABLS]*E[n]$

$= [I(s) - I(r)]*(\text{BL pure premium}).$

Mathematically, the expected value pure premium for an excess contract is the difference in expected value pure premium of two "first dollar" contracts.

The solution to the problem requires two adjustments to be made:

- *Adjust the basic limits severity due to inflation*
- *Use the new trended ILFs*

The average excess pure premium for the layer of $2,000,000 excess of $500,000 is equal to:

$\{[I(s/a) - I(r/a)] \div I(b/a)\} * a*E[g(x,s/a)]*E(n)$

$= \{[I(2{,}500{,}000/1.05) - I(500{,}000/1.05)] \div I(100{,}000/1.05)\} *1.05*(16{,}000*.956)*(.20)$

$= \{[2.304 - 1.600] \div .956\} *3{,}212.16 = 2{,}365.44$

Question 6 – Model Solution 2 *(part b only)*

We are given the following in the problem: a = 1.05 ; freq. =.20 ; E[g(x;100,000)] = 16,000; b =100,000

To calculate the pure premium charge adjusted for one year of inflation for a policy covering losses in the layer $2,000,000 excess of $500,000, compute the following:

1. *Pure premium for the layer 2,000,000 xs 500,000 (prior to inflation)*
2. *Inflation in the layer 2,000,000 xs 500,000*

Before inflation:

PP@100,000 = (16,000)(.2) = 3,200

PP@500,000 = (3,200)(1.632) = 5,222

PP@2,500,000 = (3,200)(2.325) = 7,440

Therefore, PP for 2,000,000 xs 500,000 = 7,440 – 5,222 = 2,218

*Recall that the average increase in **excess losses** with a **fixed** upper limit:*

$$\frac{E[h(x';r,j)]}{E[h(x;r,j)]} = \frac{E[g(x',s)]-E[g(x',r)]}{E[g(x,s)]-E[g(x,r)]} = a*\frac{E[g(x,s/a)]-E[g(x,r/a)]}{E[g(x,s)]-E[g(x,r)]} = a*\frac{I(s/a)-I(r/a)}{I(s)-I(r)}$$

*Thus, inflation in layer = a * [I(s/a) - I(r/a)] / [I(s) - I(r)]*

*=1.05 * [I(2,500,000/1.05) -I(500,000/1.05)] / [I(2,500,000) -I(500,000)]*

*=1.05 * (2.304 -1.600) / (2.325 -1.632) = 1.06667*

Therefore, the inflation adjusted layer pure premium = (1.06667)(2,218) = 2,366

Supporting materials. An excerpt from our online note cards
Excess of Loss Coverage
Let h(x;r;j) represent the excess of loss cost function, for losses > r (the retention), up to j (the maximum liability), and s=r+j.

_E[h(x;r;j)] = E[g(x;s)] - E[g(x;r)_

Excess of Loss pure premium = E[h(x;r;j)]*E[n]
= [I(s) - I(r)]*[ABLS]*E[n]
= [I(s) - I(r)]*(BL pure premium).

Mathematically, the expected value pure premium for an excess contract = the difference in expected value pure premium of two “first dollar” contracts.