

**Questions from the 2007 Exam:**

5 (3 points) An insurance company uses two variables for pricing its auto insurance product—Gender and Territory.

The ratemaking actuary has decided to use the following data, together with a multiplicative minimum bias model and the balance principle, to develop new classification rates.

Exposure Units

	Y1 (Male)	Y2 (Female)
X1 (Territory 1)	2,000	3,000
X2 (Territory 2)	2,500	4,000
X3 (Territory 3)	1,000	2,000

Observed Loss Cost Relativities

	Y1 (Male)	Y2 (Female)
X1 (Territory 1)	4.0	6.0
X2 (Territory 2)	2.0	5.0
X3 (Territory 3)	3.0	1.0

The initial values for territorial relativities are:

$X1 = 3.0$

$X2 = 2.0$

$X3 = 1.5$

Calculate the relativities for both Gender and Territory after one complete iteration.

**Supporting materials. Our solution to question 24 from the 2003 Exam.**

24. (5 points) You are given the following losses and exposures. Assume the base class is Good Protection Class with a New Roof ( $x_1, y_1$ ). Assume initially that  $y_1 = 1.00$  and  $y_2 = 1.30$ .

Total Losses

Age of Roof	Protection Class		
	Good ( $x_1$ )	Average ( $x_2$ )	Poor ( $x_3$ )
New ( $x_1$ )	100,000	255,000	60,000
Old ( $x_2$ )	360,000	650,000	200,000

Exposures

Age of Roof	Protection Class		
	Good ( $x_1$ )	Average ( $x_2$ )	Poor ( $x_3$ )
New ( $x_1$ )	200	500	100
Old ( $x_2$ )	600	1,000	200

a. (2.75 points) Calculate the first iteration of relativities for Protection Class and Age of Roof, using the Maximum Likelihood Multiplicative Model (Balance Principle).

**Supporting materials. Our solution to question 24 from the 2003 Exam.**

**Solutions to questions from the 2003 Exam:**

24. (5 points) You are given losses and exposures. Assume the base class is Good Protection Class with a New Roof ( $x_1, y_1$ ). Assume initially that  $y_1 = 1.00$  and  $y_2 = 1.30$ .

a. (2.75 points) Calculate the first iteration of relativities for Protection Class and Age of Roof, using the Maximum Likelihood Multiplicative Model (Balance Principle).

*Under the maximum likelihood multiplicative model (MLMM), the  $x_i$  relativity is computed as follows:*

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j}, \text{ where (in this problem) the } n_{ij} \text{ is the exposures for age of roof } i \text{ and protection class } j;$$

and  $r_{ij}$  is the loss cost relativities. To calculate the  $r_{ij}$ , 2 steps are involved:

1. Calculate a matrix of frequencies (Loss/Exposure)
2. Calculate a matrix of relative frequencies (frequencies / base class frequencies). The problem states that the base class is Good Protection Class with a New Roof ( $x_1, y_1$ ).

Step 1: Compute a matrix of frequencies (Loss/Exposure)

<b>Compute</b>			
<b>1. Actual Claim Frequencies [ 1. ÷ 2.]</b>			
Age of Roof	Protection Class		
	Good ( $x_1$ )	Average ( $x_2$ )	Poor ( $x_3$ )
<b>New</b>	500	510	600
Old	600	650	1000

Where 1. is losses (given) and,  
2. is exposures (given)

Step 2: Compute a matrix of relative frequencies (frequencies/base frequencies)

<b>2. Frequency Relativities [ 1. ÷ 1 base class.]</b>			
Age of Roof	Protection Class		
	Good ( $x_1$ )	Average ( $x_2$ )	Poor ( $x_3$ )
<b>1</b>	1.000	1.020	1.200
2	1.200	1.300	2.000

Step 3: Using the formula for MLMM, and the results from steps 1 and 2, compute the first iteration of relativities for Protection Class and Age of Roof

$$x_1 = \frac{(n_{11}r_{11} + n_{12}r_{12}) \div (n_{11}y_1 + n_{12}y_2)}{= \frac{(200 * 1.0 + 600 * 1.2)}{(200 * 1.0 + 600 * 1.30)} = .939}$$

$$x_2 = \frac{(n_{21}r_{21} + n_{22}r_{22}) \div (n_{21}y_1 + n_{22}y_2)}{= \frac{(500 * 1.02 + 1,000 * 1.30)}{(500 * 1.0 + 1,000 * 1.30)} = 1.006}$$

$$x_3 = \frac{(100 * 1.2 + 200 * 2.0)}{(100 * 1.0 + 200 * 1.30)} = 1.444$$

Step 4: Using the results from Steps 1 – 3, and the given information in the problem, compute the following:

$$y_1 = \frac{(200 * 1.0 + 500 * 1.02 + 100 * 1.20)}{(200 * .939 + 500 * 1.006 + 100 * 1.444)} = \frac{830}{835.20} = .994$$

$$y_2 = \frac{(600 * 1.2 + 1,000 * 1.30 + 200 * 2.00)}{(600 * .939 + 1,000 * 1.006 + 200 * 1.444)} = \frac{2,420}{1,858.20} = 1.302$$

**Supporting materials. An excerpt from our online notecards**

*THE BALANCE PRINCIPLE*

*The balance principle requires that (after optimizing the relativities): the sum of the indicated pure premiums equals the sum of the observed loss costs along every row and column.*